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DRAFT: INVENTING A NEW METHOD IN STATICS THROUGH KNOWLEDGE IN KINEMATICS

Offer Shai*
School of Mechanical Engineering
Tel Aviv University

Yoram Reich
Department of Mechanical Engineering
Tel Aviv University

ABSTRACT
Suppose we designed an innovative structure, such as a deployable tensegrity structure, and suppose that while doing so, we are faced with a problem for which no available method could solve. The problem is such that if left without solution could hamper the further development of this structure. Infused Design (ID) is a method for transferring knowledge including solutions and methods between diverse disciplines. If an appropriate method is missing from a particular discipline, ID could be used to invent a new method by transforming methods from other disciplines. This paper reports on inventing a new method that allows finding relations between forces in a tensegrity structure thus determining which element would be a strut and which a cable. The invention process is described in detail. As an approach that has proved useful in inventing several design methods, ID constitutes a fertile source for extending our engineering and scientific knowledge.

1. INTRODUCTION
The process of solving difficult problems often presents numerous difficult steps. Two of them are the generation of an idea that might solve the problem, and the ability to realize the idea. Sometimes an idea exists but there is no easy way to realize it or its further development requires unavailable knowledge. This paper deals with a situation in which a novel type of structure, *adjustable, deployable tensegrity structure*, is developed for various applications, while the knowledge for realizing it is only partially known. The missing knowledge risks the whole endeavor. Therefore, needs to be invented or discovered in some way.
Tensegrity structures (**Error! Reference source not found.**) consist of two types of elements: struts and cables. A strut can sustain only compression loads and a cable only tension. One of the main difficulties in using these advanced structures is the synthesis problem called *form finding* **(**Tibert and Pellegrino, 2003**)**. In this problem you are given (1) the elements of the structure, the cables and the struts, and (2) the topology of the structure, i.e., which element is connected to which, and you have to find a configuration in which the tensegrity structure is rigid. This problem is very difficult, since for even slight changes in the configurations, the tensegrity structure might lose its rigidity. There has not been a method for addressing this design problem until recently. This paper reports on the discovery of a method that was found to be very useful in solving this problem. The paper focuses on the process of inventing this new method in statics, by transforming it from another engineering domain, from kinematics. Two new entities in statics are widely used in the invented method: *face force* and *equimomental line* (Shai and Pennock, 2006; Shai *et al.*, 2009b). It is interesting to note that the two new entities that were revealed in the last five years were also invented in a way similar to the process of inventing the proposed method.

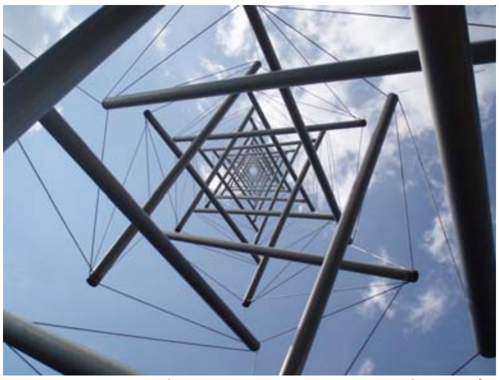

Figure 1. Tensegrity structures: Kenneth Snelson's Needle Tower: An example of a tensegrity structure. This picture is licensed under the Creative Commons Attribution-ShareAlike 2.5 Netherlands

* Senior Lecturer and author of correspondence, Phone: (972) 640-7385, Fax: (972) 640-7617, Email: shai@eng.tau.ac.il.

The driver of both inventions is Infused design (ID), which is an approach for transferring knowledge between disciplines including solutions and methods (Shai and Reich, 2004a,b). ID has been used for designing creative designs (Shai 2005a,b; Shai *et al*., 2009a) and for inventing other design methods (Reich *et al*., 2008a).

The remainder of the paper reviews ID (Section 2); details the process of discovering the new method (section 3); and provides conclusions and potential of the ideas presented in this paper (section 4).

2. ID: THE DRIVER OF INVENTIONS

Infused Design (ID) is a method that rests on a solid mathematical foundation for combinatorial representations of systems (Figure 10). ID has demonstrated new forms of creativity by generating designs that were not conceived before, by studying and transferring across disciplines, designs from seemingly unrelated disciplines (Shai 2005a,b; Shai *et al*., 2009a)).

The representations that are the foundation of ID are discrete mathematical models, called graph representations; they include Resistance Graph Representation (RGR), Potential Graph Representation (PGR), Flow Graph Representation (FGR), and others. These representations can represent diverse systems, e.g., RGR is isomorphic representation of both electrical circuits and indeterminate trusses (Shai, 2001b). These representations and their relations (see Figure 10), such as the duality between PGR and FGR allow for transforming automatically one representation to others connected to it (Shai, 2001a). Such transformations can be done automatically (Shai *et al*., 2009a). The automated transformation in ID is provably mathematically correct as these transformations are guaranteed to produce the same behavior for the original and transformed representation. In this way, ID is different from other creativity support methods such as TRIZ that only guide towards new designs without guarantee.

To better illustrate the process, consider the following example assuming that the disciplines that participate in the design are all modeled in Figure 10; a discipline that is still not represented cannot participate in the process. Members of the multidisciplinary team start by using their customary disciplinary model and terminology for each discipline, e.g., PGR for mechanisms and FGR for static systems. In order to integrate all the disciplinary representations they need to traverse the map of representations to find one common representation that accommodates all the original representations. For this particular example, according to Figure 10, PGR, FGR and RGR could serve as the common representation because PGR and FGR are dual and because RGR is more general to both.

Once the common representation is found, there is a path in the representations map that allows transferring knowledge from one discipline to the other. This knowledge includes solutions or solution methods.

ID presents opportunities that extend beyond this description such as bootstrapping effects. For example, through the interaction with mathematicians, the transfer of knowledge between kinematics and mathematics, led to the generation of new knowledge in mathematical rigidity theory that in turn, led to a new method for determining the rigidity of structures (Reich *et al*., 2008a). The process through which close interaction leads to a stepwise development in both disciplines has bootstrapping properties.

The particular method discovery process in this paper follows the path outlined in Figure 11. A problem arose when designing tensegrity structures, is transformed into abstract graph representation. At that level, search through the map of representations is performed through related representations to connected disciplines to find a corresponding method or one that is closely related. A corresponding method is found in mechanisms and transformed back into statics by also using concepts brought from two other representations.

While this process may sound similar to analogical reasoning or to ideas underlying methods such as TRIZ, it is completely different. One important difference is the fact that all other methods provide ideas that may not work once tried, while the use of ID guarantees the success of the transformation, making it possible to fully automate. Another difference is that the analogical transformations in ID are founded on well established proven mathematical theorems from discrete mathematics that guarantee the soundness of the transformations.

3. THE DISCOVERY PROCESS

In this section we introduce a novel method in statics that enables finding the relation between the value of any two forces in rods without the need of knowing the value of each force and without writing all the equilibrium force equations and solving them.

This method could be used in various applications for systems with many elements. Some of the applications can be for systems that change continuously where the force in some is easy to calculate while for others it is difficult. By finding a direct relation between forces in different elements, the structure becomes like a control system in which some elements for which it is easy to know their forces define the forces acting in other elements. A practical example of the invented method in tensegrity appears in the paper.

3.1 Preview of the new method

We start by providing a brief introduction about the use of the method for 'form finding' in 2D tensegrity structures.

It is easy to verify that for joints of degree three, i.e., the number of elements incident to a joint is three, there are two rules that well-define the type of the elements, ensuring that the joint is rigid. The rules are *arrow* and *Y* rules, defined as follows:

The *arrow* rule – If all the elements are on one side of a line that passes through the joint then the two outer elements are of the same type and the inner element is of the other type. Example of the two different cases appears in Figure 2(a1, a2).

The *Y* rule – if it is impossible to draw a line through the joint so that all the elements are on one side of the line, then all are of the same type: all are cables or struts, as appears in Figure 2 (b1, b2).

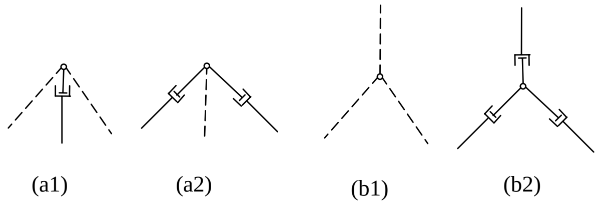

Figure 2. Example of applying the arrow and Y rules.

When the degree of the end joints of an element are both greater than three, there is no such rule and it is impossible to define the type of the element only from the geometry relation between the elements incident to the joint.

In the new method, the following procedure is used, see Figure 3. First, we find a line upon which the moment of the elements from some joint of degree three and from a joint with degree greater than three exert the same moment. Note that the two joints do not need to belong to the same element, but could be situated in different places in the structure. Let us call the aforementioned line the control line.

Now, through this control line we can find the type of the needed element as shown in Figure 3, for example, the strut incident to joint A with degree three defines the type of the element, designated by a bold line, whose two end vertices are with degree greater than three.

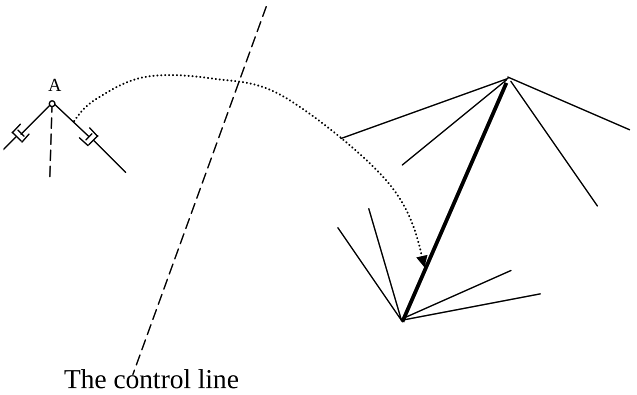

Figure 3. The dependence between the element of arrow joint and an element through the control line.

3.2 Preliminaries

In inventing this method, we employ two new entities in statics that were also invented based on the knowledge of kinematics. The first entity is the *Face Force*, for brevity we designate it by 'FF', a virtual force that exists in each face. The values of the two FFs adjacent to a rod define the force acting in that rod. Although there are difficulties in measuring the face force, this force has unique physical properties, such as potential property, which is not associated with regular and known types of forces. Since it is a force, and based on the classic association of a force with a vector, the FF should apply along some line of application. In the process of finding where it acts, the second new entity was exposed – the *equimomental line*, designated in this paper by 'eqm'. This line is a new geometrical entity that exists for any two forces in the plane upon each one of its points, they exert the same moment. The line should pass through the intersection, or their continuations, of the two forces in the direction of their subtraction.

Now that we have the definition of the eqm of any two forces, we can inquire about the eqms of FFs and their physical interpretation. Since the force in the rod is defined, as stated above, by the subtraction of its two adjacent FFs and since an eqm is defined in precisely the same way, it follows that each rod is in the direction of the eqm of its two adjacent FFs.

To examine the properties of these two new entities, let us check the special case when one of the two faces adjacent to a rod is the ground (reference) face, in which there exists a face for which we can give an arbitrary value of the face force as is done in electricity where arbitrary voltage can be given to the ground junction. From the definition of these two new entities it follows that the force in that rod is equal to the FF in the non ground face and the line along this rod is actually the eqm of that FF. Thus, when one of the two faces adjacent to a rod is the ground face, we know the eqm of the other FF and it is along the line of the rod. We call the eqm of the FF, the line along which the FF acts, the 'absolute eqm' and the eqm of two FFs 'relative eqm'.

Now, that we understand the two new entities in statics, we move to the field of kinematics, look there for a method that does a similar task – finding the velocity of a joint according to known velocity of a joint at other place of the mechanism through a related common entity, for the corresponding entities in kinematics. Once found, the method in kinematics, defined with the entities in kinematics, is transformed back into statics, resulting in a new method in statics.

In kinematics, it is possible to find the angular velocity of an arbitrary link according to the angular velocity of a given link, let us say the driving link, through the properties of instant centers, using the following principle:

Suppose we have two links (Figure 4): i and j for which we know their absolute instant centers: $\vec{I}_{i,0}$ and $\vec{I}_{j,0}$, respectively. Thus, the relative instant center is the point at which both links have the same absolute linear velocity (equation (1)):

$$\vec{\omega}_{i,0} \times (\vec{I}_{i,j} - \vec{I}_{i,0}) = \vec{\omega}_{j,0} \times (\vec{I}_{i,j} - \vec{I}_{j,0}) = \vec{V}_{Ii,j/0} \quad (1)$$

Now, let us transform this knowledge into statics through the duality in

Table 1. Note that the transformation employed in the paper guarantees the correspondence between 'analogous' entities and all their associated properties, behavior and theorems.

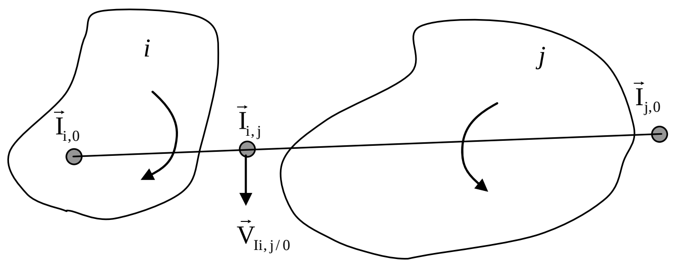

Figure 4. Three links – *i*, *j*, and the ground (designated with 0) – and their three corresponding relative instant centers. Note: the three instant centers are collinear due to the Kennedy theorem (Erdman and Sandor, 1997).

Table 1. Corresponding entities through duality

Kinematics	**Statics**
Instant center	Equimomental line
Angular velocity	Force
Linear velocity	Moment
Link	Face
Absolute angular velocity of a link	Force in the face – Face Force.
Kennedy theorem – For any three links, their three relative instant centers are collinear.	Dual Kennedy theorem – For any three faces, their three relative equimomental lines intersect at the same point.

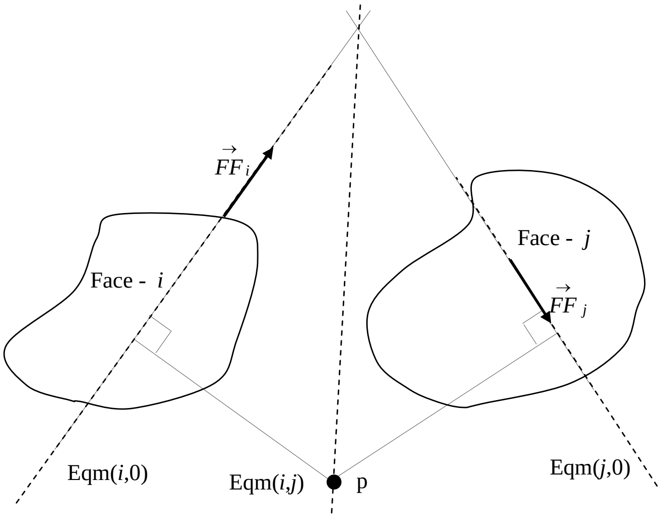

Figure 5. The relative eqm of two Face Forces.

Applying the duality transformation from Table 1 to equation (1) yields equation (2):

$$\vec{FF}_i \times (p - eqm(i,0)) = \vec{FF}_j \times (p - eqm(j,0)) = M_{i,j/P} \quad (2)$$

where:

$\vec{FF}_k$ – face force vector in face *k*.
eqm(*x*,*y*) – the equimomental line of faces *x* and *y*.
p – a point on the equimomental line *eqm(i,j)*.

$M_{i,j/p}$ – the moment exerted by the two face forces $\vec{FF}_i$ and $\vec{FF}_j$ at point *p*.

3.3 Example problem statement: deciding the type of an element in a tensegrity structure

Structures such as the one shown in Figure 1 are based on the principle of tensegrity in which struts (compression elements) and cables (tension elements) serve together to generate a stiff structure. Tensegrity serves as the basis for advanced structures including deployable structures (Figure 6) and has significant future applications.

When synthesizing complex tensegrity structures, it becomes important to decide which element would be a strut and which a cable. The choice depends on whether the element is in compression or tension.

Figure 6. Deployable tensegrity structure. This structure was developed at the adjustable deployable tensegrity structure lab at Tel Aviv University.

Suppose we need to decide the type of the element AC in a tensegrity structure, i.e., to decide whether it is a cable or a strut. Such decision is not straightforward. Let us use the equimomental lines and face forces to solve the problem.

3.4 The principle of deciding the type of the element in tensegrity structure

Suppose we want to know the type of element (*x*,*y*) adjacent to two faces FF_R and FF_L as shown in Figure 7. In this section, we show that it is enough to know at least one face force, according to which all the *FFs* can be determined using the relative equimomental lines. Let us designate the known face force by FF_z.

Applying Equation (2) enables to determine as follows:

$$\vec{FF}_R = \frac{(p - eqm(Z,0)) \times \vec{FF}_Z}{(p - eqm(F_R,0))} \quad (3)$$

where (*p*-*eqm*(*Z,W*)) stands for the distance vector between point *p* and the equimomental line *eqm*(*Z,W*)). The force in face F_L, , is determined by the same equation, but this time the denominator is (*p*-*eqm*($\underline{F_L}$,0)).

Now that we know the two face forces adjacent to element (*x*,*y*) it is possible to calculate the force acting in the element using the following rule.

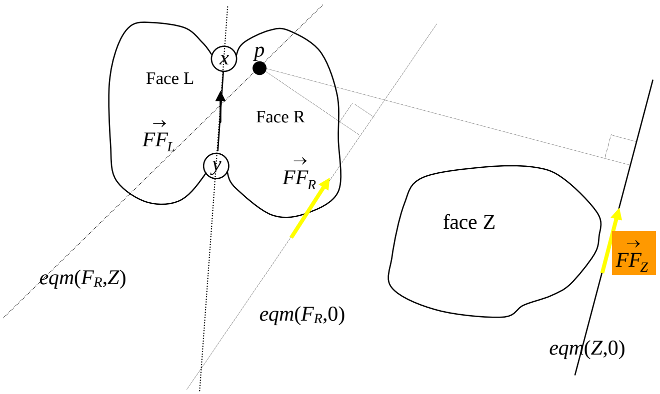

Figure 7. Description of the three equimomental lines needed to determine $\vec{FF}_R$ from $\vec{FF}_Z$

3.5 Rule for computing the force in the element from its two adjacent face forces

Given an element '*e*' and its two adjacent face forces, the force acting in this element is computed by subtracting the left from its right face force:

$$\vec{f}(e) = \vec{FF}_R - \vec{FF}_L \quad (4)$$

From equation (4) it follows that once all the face forces are known then the forces in all the rods can be computed by subtracting the values of the two adjacent *FFs*.

From equation (4) it is shown below that for any two *FFs* adjacent to an element, their projection onto a line perpendicular to the element should be equal, as given in the following property.

3.6 The perpendicular projection property of *FFs*

Let FF_R and FF_L be the right and left face forces of element *e*. Their projections on the line perpendicular to *e*, designated by the unit vector $\hat{n}_e$, should be equal (see also Figure 8).

Proof: Multiplying equation (4) by the perpendicular unit vector, $\hat{n}_e$, and since $\vec{f}(e) \bullet \hat{n}(e) = 0$, then after rearranging terms we derive:

$$\vec{FF}_R \bullet \hat{n}_e = \vec{FF}_L \bullet \hat{n}_e \quad (5)$$

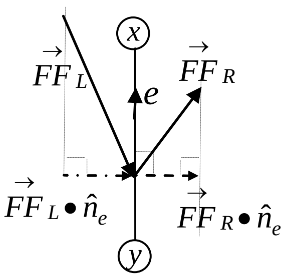

Figure 8. The projection perpendicular property of two adjacent face forces.

Now, based on the above rule and property, we are ready to introduce a method for deducing the type of the element from its two adjacent *FFs*.

3.7 The decision rule of the element type from the adjacent *FFs*

In a previous publication (Shai, 2001a) it was shown that when the sign of the scalar force is plus there is a compression force, and otherwise tension. These signs define the types of elements, i.e., if the sign is plus then the element should be a strut and otherwise a cable. In this paper we define the sign of the element, i.e., the type of the element through the face forces, an entity that was not known at that time and is the fundamental entity in the invented method. Coming to decide about the sign of an element in 2D, we should take into account two types of signs: topology and geometry, as is done in representing diverse engineering systems in 2D.

The topology sign is obtained by making the element directed which defines which face will be on its left/right side and the head joint. The physical interpretation of the topology direction indicates that the geometry force in the element, explained below, is acting upon the head joint.

The geometry sign is derived from the following equation that defines the vector force acting in the element as follows:

$$\vec{f}(e) = f(e)\hat{i}(e) \quad (6)$$

where $\vec{f}(e)$ is the vector force in the element, *f*(*e*) the scalar force and the unit vector in the direction of the element (see Figure 9). If the force vector is directed towards the head vertex, it means that the force acting in the rod upon the head joint is outside, i.e., there is a compressive force in the element, thus it will be a strut, otherwise, it is tension and will be a cable. From this relation, it follows that if the scalar product of the force in the element by the unit direction vector of the element is positive (negative) then the element is a strut (cable).

To simplify the process of defining the faces and overcome the limitation that the face forces are limited to planar graphs, we employ the known Bow's notations, a method used also by Maxwell (Maxwell, 1876).

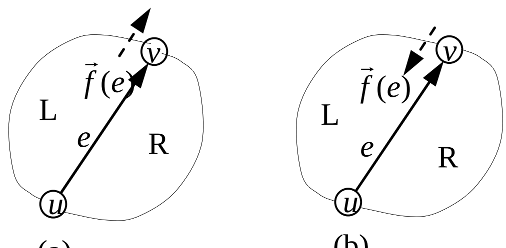

Figure 9. Description of the relations between the force vector and the topological direction of the element. (a) The element and the force vector are in the same direction thus the element should be a strut; (b) opposite direction means the element should be a cable

4. DISCUSSION

We demonstrated how a new method, enabling to decide the type of a tensegrity element according to other known element, that supports the design of new structures, could be invented. Such invention is made possible by ID, a method originally conceived to transfer solutions and methods known in one discipline to a method in another discipline. In several cases, as appears in this paper, we invented a new method for tensegrity structures that was previously unknown in other disciplines. These inventions required collecting different pieces of knowledge including the invention of new disciplinary concepts such as the face force and the equimomential line. Once such method is invented, it could be transferred to other disciplines, potentially leading to new interesting capabilities. What the present example demonstrates is that complex methods could be invented by thinking inside the disciplinary knowledge, in this case in statics, and bringing from the outside (other disciplines) pieces of knowledge, in this case kinematics, that are missing. Partial methods that seem useless or incomprehensible in one discipline could be viewed as highly useful in another. ID allows bringing together multiple perspectives from different disciplines in a formal way to integrate them into coherent methods.

The scope of creative design thinking extends much beyond solving particular design problems. It relates to our own practice as design researchers. Design support methods such as ID could be used in our own research practice to bring together multiple perspectives from different disciplines to solve complex problems. One example of such collaborative discovery was reported before (Reich *et al*., 2008) and the present study provides another result, which is more profound since it uses new concepts that have been invented also by making use of ID.

Similar to problems that are pending in mathematics or in computing, it would be interesting to create a database of such unsolved problems in engineering and try to approach them with methods such as ID.

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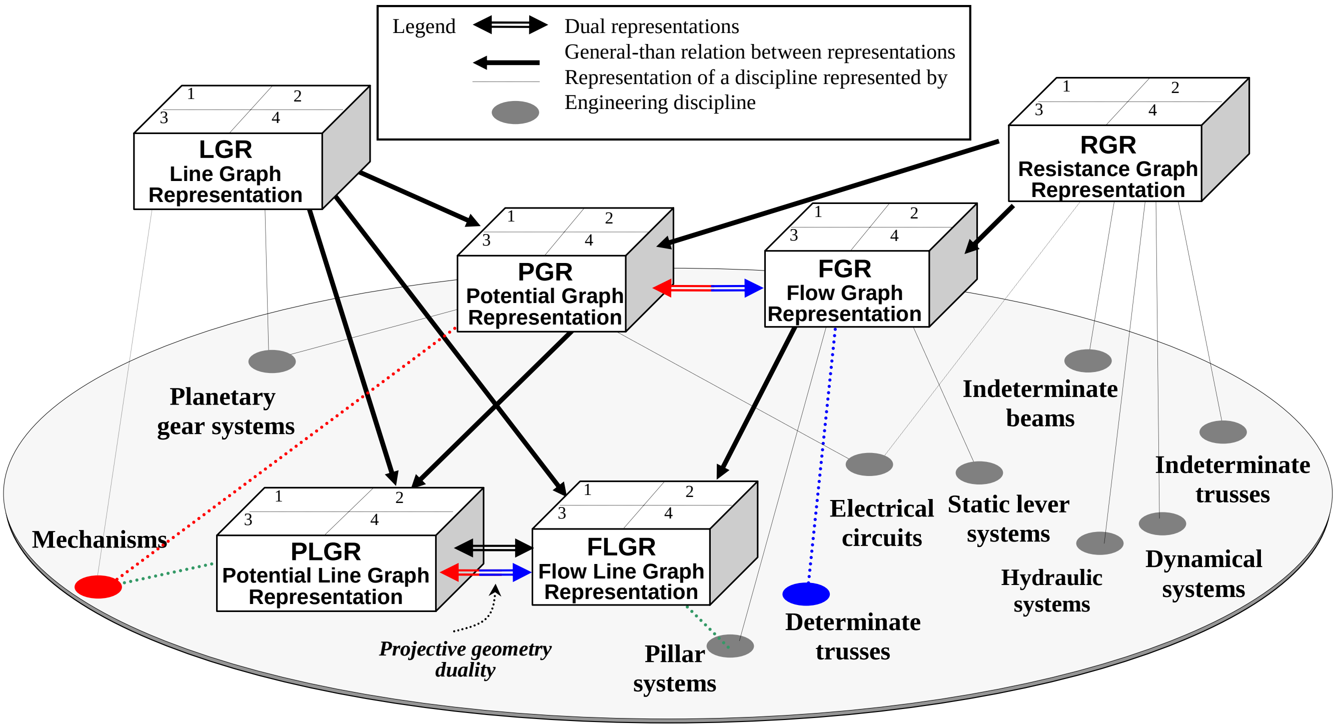

Figure 10: Map of graph representations, their interrelations, and association with engineering systems (Shai and Reich, 2004b). The map includes representations and the engineering domains to which they have already been applied. The four representations: PGR, FGR, FLGR and PLGR have been used for deriving the material appearing in this paper by transforming knowledge and methods from mechanism into statics – determinate trusses.

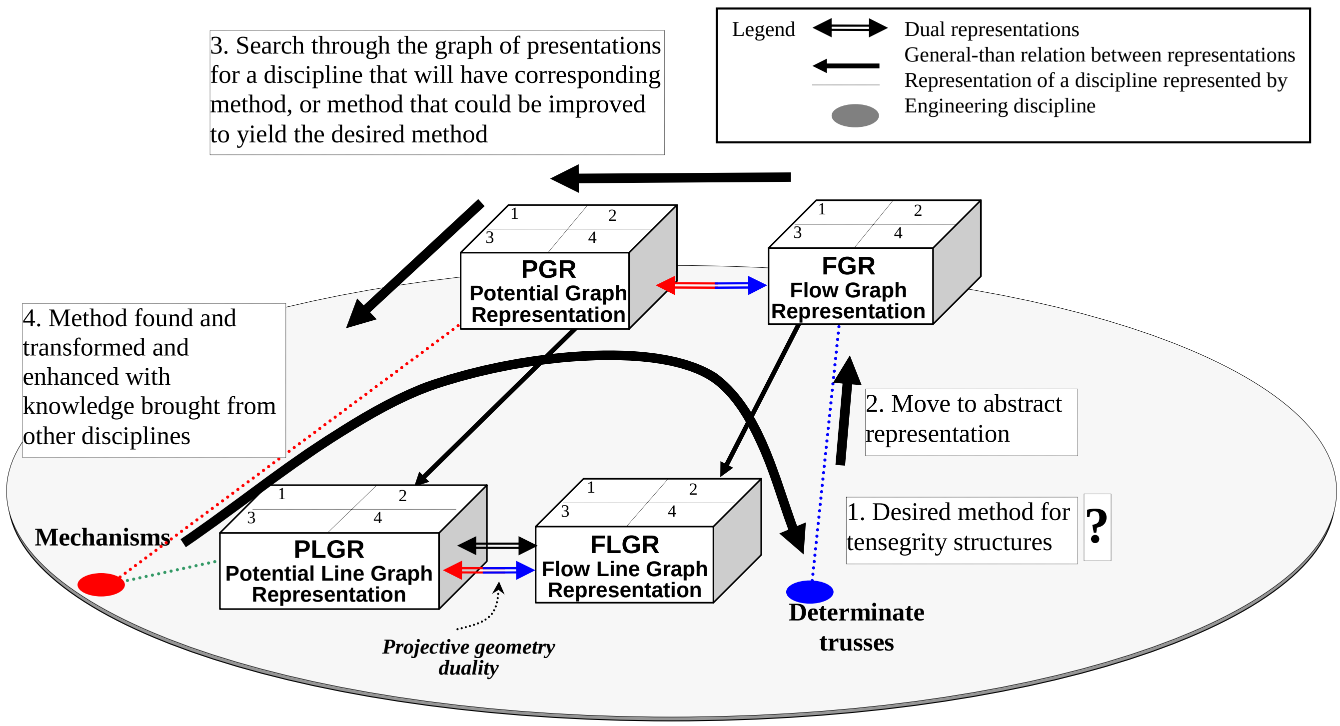

Figure 11: The path in the map of graph representations, followed for inventing the new method